1. INTRODUCTION
The permanent-magnetsynchronous motor (PMSM) has received notorious acceptancein industrial applications due to its high efficiency, hightorque-current ratio, low noise, and robustness [1][2][3]. In particular, the interior PMSM (IPMSM) provides a smooth rotorsurface and better dynamic performance [4]. In order to make appropriate use of these relevant advantages in this machine, the use and application of an accurate and efficient control technique needs to be developed and tested; this precisely the purpose of this paper. This goal will be achieved with the use of the Maximum Torque per Ampere (MTPA) strategy [5] and vector control theory.

For simulation and analysis purposes of the IPMSM control, common electrical parameters for this kind of machine analyzed by references [6] [7] will be assumed as Table 1 details.

2. VECTOR CONTROL USING MTPA TECHNIQUE
The synchronous reference frame will be implemented by the use of Park transformations [8] so that we can transform a 3-phase system into dq components (Equation 1). This will be especially helpful when controlling stator currents because instead of tracking three 50/60 Hz sinusoidal reference signals (Ia, Ib, Ic) only two DC reference signals will need to be tracked (the quadrature and direct currents, Iq and Id respectively). On the other hand, dq to abc transformation (See Equation 2) is useful when feeding back voltage references to the machine as three phase values are again required.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOL</th>
<th>VALUE</th>
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<tbody>
<tr>
<td>Stator resistance</td>
<td>Rs</td>
<td>0.43 Ω</td>
</tr>
<tr>
<td>D-axis stator inductance</td>
<td>Ld</td>
<td>27 mH</td>
</tr>
<tr>
<td>Q-axis stator inductance</td>
<td>Lq</td>
<td>67 mH</td>
</tr>
<tr>
<td>No-load peak line-to-line voltage constant</td>
<td>Vpk/krpm</td>
<td>98.67 V (peak value @ 1000rpm)</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>p</td>
<td>2</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>J</td>
<td>0.00179 kg*m²</td>
</tr>
<tr>
<td>Rated current</td>
<td>In</td>
<td>10A</td>
</tr>
<tr>
<td>Mechanical Load Torque</td>
<td>K1</td>
<td>0.00764 Nm/(rad/s)</td>
</tr>
</tbody>
</table>
To successfully achieve the MTPA condition, the synchronous reference frame with the dq axes placed like in Fig. 1 will be considered. There we can see that the q axis is ahead the a axis with the θ angle (θe = θ). Additionally, it can be observed the q and d axes being in quadrature and rotating at an angular speed equal to We. Therefore in this quadrature reference, the current only has direct (Id) and quadrature (Iq) components.

\[
\begin{bmatrix}
V_{qs} \\
V_{ds} \\
V_{c}\end{bmatrix} =
\begin{bmatrix}
\frac{2}{3} \cos(\Theta) & \frac{2}{3} \cos(\Theta - 2\pi/3) & \frac{2}{3} \cos(\Theta + 2\pi/3) \\
\frac{2}{3} \sin(\Theta) & \frac{2}{3} \sin(\Theta - 2\pi/3) & \frac{2}{3} \sin(\Theta + 2\pi/3) \\
1/3 & 1/3 & 1/3
\end{bmatrix}
\begin{bmatrix}
V_{as} \\
V_{bs} \\
V_{cs}\end{bmatrix}
\]

(1)

\[
\begin{bmatrix}
V_{as} \\
V_{bs} \\
V_{cs}\end{bmatrix} =
\begin{bmatrix}
\cos(\Theta) & \sin(\Theta) & 1 \\
\cos(\Theta - 2\pi/3) & \sin(\Theta - 2\pi/3) & 1 \\
\cos(\Theta + 2\pi/3) & \sin(\Theta + 2\pi/3) & 1
\end{bmatrix}
\begin{bmatrix}
V_{qs} \\
V_{ds} \\
V_{c}\end{bmatrix}
\]

(2)

For the assumed dq scheme, the torque and current in a PMSM are related with Equations 3 to 6 [8][9]:

\[
T = 1.5 P (\lambda_{pm} I q + (Ld - Lq) I d I q)
\]

(3)

\[
I q = I . \cos(\varphi)
\]

(4)

\[
I d = -I . \sin(\varphi)
\]

(5)

\[
I = \sqrt{I q^2 + I d^2}
\]

(6)

Where \(\lambda_{pm}\) is the amplitude of the permanent magnet machine flux linked to the stator windings, while LD and LQ are the direct and quadrature inductances respectively. By using Equation 7 [10] and considering the figures in Table 1, for this study case, \(\lambda_{pm}\) is found to be equal to 0.272 [V.s].

\[
\lambda_{pm} = \frac{60 \times V_{pk}/k_{rpm}}{2 \times \sqrt{3} \times \pi \times P \times 1000}
\]

(7)

Contraction to the Surface Permanent Magnet Synchronous Machine (SPMSM) [9][11] where the direct current is set to be null, in the IPMSM the direct and quadrature currents need to be both appropriately controlled to produce the desired current module (I) and thus the required torque. However, there is an infinite number of Iq and Id currents combinations which may produce the same final current module as Equation 6 implies. This is reason why the MTPA control strategy seeks to maximize the torque for a given amount of current by maximizing the torque function with respect to the \(\varphi\) angle \((\frac{d}{d\varphi}T = 0)\). If we do this after replacing Equations 4 and 5 in 3, we have:

\[
\frac{3}{2} P (-\lambda_{pm} I q \sin(\varphi) - I^2 (Ld - Lq) \cos(2\varphi)) = 0
\]

(8)

When solving Equation 8 for \(\varphi\) and considering the nominal current \(I = \text{nominal} = 10\) [A] (Table 1); from the obtained mathematical answers the angle that produces the biggest torque is \(\varphi = 33.86^\circ\) with 12.32 [Nm]. For this condition and using Equations 4 and 5, we obtain \(I q = 8.303\) [A] and \(I d = -5.57\) [A].

In Fig. 2 it can be visualized the Id and Iq components relation in MTPA conditions for different current amplitudes (I) between 1 and 10 [A]. As we can see, the bigger Iq the bigger the negative Id. As permanent magnets present higher reluctance than iron, the inductance along the d axis tends to be smaller than the one on the q-axis [15]. For these reason in typical IPMSMs, \(Ld\)s lower than \(Lq\). This results on the need of introducing negative Id to produce positive torque on the d axis.
Where $We$ and $Wr$ are the electrical and mechanical speeds of the rotor respectively. If we plot $EMFd$ vs $EMFq$, it can be observed in Fig. 3 the inverse relation between their amplitudes. The bigger the $EMFq$, the lower the $EMFd$ module. This can be explained as the $EMFd$ depending on q-axis variables and vice versa, as Equations 9 and 10 imply.

To implement the MTPA strategy, Equation 6 is replaced in 3 so that we can get $Id$ in function of $Iq$ as we can see in Equation 12.

$$Id = -\frac{\lambda_{pm}}{2(Ld-Lq)} - \sqrt{\frac{\lambda_{pm}^2}{4(Ld-Lq)^2} + Iq^2}$$  \hspace{1cm} (12)

Then, replacing Equation 12 in 3 we finally have:

$$T = \frac{3}{2} P \left[ \frac{1}{2} \lambda_{pm} Iq - (Ld-Lq) \sqrt{\frac{\lambda_{pm}^2 Iq^2}{4(Ld-Lq)^2} + Iq^2} \right]$$  \hspace{1cm} (13)

Now, we need to bear in mind that the reference torque is the input of the system (as it depends from the machine’s load) while $Iq$ is the output. As in the previous Equation it is not simple to mathematically isolate $Iq$, we can give appropriate values to $Iq$ to get their corresponding torque $T$ components. If we do this we can plot $Iq$versus $T$ as in Fig. 4 so that we could use a curve fitting method [12] to finally obtain an expression for $Iq$ in function of the required torque $T$. To get enough accuracy when doing this procedure, a fifth polynomial degree curve for the fitting process was used (Equation 14). This mathematical approximation model presented a 95% of confidence bounds with a Root Mean Squared Error (RMSE) of 1.65%. In fact, the fitting curve is very accurate as we can clearly observe in Fig. 5 where a deep zoomed area from the non-linear part of the original plot (Fig. 4) has been made.

$$Iq = a.T^5 + b.T^4 + c.T^3 + d.T^2 + e.T + f$$  \hspace{1cm} (14)

<table>
<thead>
<tr>
<th>CONSTANT</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.448E-6</td>
</tr>
<tr>
<td>b</td>
<td>-0.0001314</td>
</tr>
<tr>
<td>c</td>
<td>0.00469</td>
</tr>
<tr>
<td>d</td>
<td>-0.08753</td>
</tr>
<tr>
<td>e</td>
<td>1.249</td>
</tr>
<tr>
<td>f</td>
<td>0.02338</td>
</tr>
</tbody>
</table>

However, it must be mentioned that for the MTPA technique to work properly, the parameters of the machine ($\lambda_{pm}$, $Ld$, $Lq$) should not vary on time [13][14]. This is why it is very critical to provide nominal operation conditions for the motor as well to successfully control its temperature. Lastly, for a given required torque only $Id$ is missing and it can be obtained by means of Equation 12.
3. TORQUE CONTROL TOPOLOGY

The inner loop shown in the block diagram detailed in Fig. 6 has been used to perform the torque control of the IPMSM. The implementation of the torque control using PSIM® Software, including the control and power stages, is the one exposed in detail in Fig.s 7 and 8.

For the design of the $I_q$ and $I_d$ regulators, the selected closed loop bandwidth was 100 [Hz]. The closed loop structure is shown in Fig. 9, being $L$ and $R$, the per-phase inductance and resistance of the motor. However, for the $I_q$ and $I_d$ regulators we consider $L_q$ and $L_d$ respectively for the design of the PI controllers in Equations 15 and 16. Thus, we get for the $I_q$ controller $K_p=42.09$ with $K_i=6.41$, while for the $I_d$ controller it was attained $K_p=16.96$ and $K_i=15.92$.

Additionally, if we plot a Bode diagram for the system as in Fig. 11, it can be checked that with a magnitude of -3 [dB] the closed loop frequency response of the system is equal to 99.7 [Hz] which is again in a practical way the required value.
This fact again corroborates an appropriate design of the controllers.

In order to briefly test if the MTPA technique was accomplished, we could simply change the phase angle advance ($\varphi$) for the current. In MTPA condition, when 10 [Nm] is required, $I_q = 7.281$ [A] and $I_d = -4.635$ [A]. This implies a total current of 8.631 [A] if we use Equation 6, this represents an advance angle $\varphi = 32.48^\circ$.

With the same amount of total current (8.631 [A]), if we change for example the angle $\varphi$ to 50°, the currents are $I_q = 5.54$ [A] and $I_d = -6.61$ [A]. Replacing these values as well as the initially considered parameters of the machine in Equation 3, we obtain that the torque is equal to 8.91 [A]; which is a lower value than the one obtained with MTPA (10 [Nm]). It can be verified for different $\varphi$ angles that the only one producing MTPA condition was the one achieved by means of the detailed methodology.

On the other hand, when talking about the response of the system, it must be mentioned that initially, the back-emf effect was neglected in the controllers. However, this produced an unwanted response on $I_d$ especially as Fig. 12 reveals. After implementing back-emf compensation for the $I_q$ and $I_d$ current controllers, the $I_d$ current and torque responses were improved as Fig. 13 reveals.

4. SPEED CONTROL LOOP FOR THE IPMSM

Once the current controllers behave properly, the speed control loop (outer loop in Fig. 6) will be implemented with the same procedure used for the current regulators. The closed loop system has the same configuration as the one in Fig. 9, however this time the transfer function of the plant to be controlled is:

$$G(s) = \frac{1}{(J+J_{load})s+(B+K1)}$$

Where, $J$ and $J_{load}$ are the machine’s and load’s moment of inertia respectively. $J_{load}$ will be considered equal to 0.030 [kg.m$^2$] while $B$, which is the friction coefficient of the machine will obtained as Equation 19 exposes. There, $\tau_{mech}$ is the mechanical time constant of the machine and it will be assumed to be equal to 0.3 [s] which is a typical value.

$$\tau_{mech} = \frac{J}{B}(19)$$

For this case the PI constants for the speed regulator will be:

$$K_p = 2.\pi.Bandwidth_{Hz}.(J+J_{load})$$

$$K_i = (B+K1)/(J+J_{load})$$

Thus, we have $K_p=0.08765$ and $K_i=3.379$. Verifying the time response ($Tr$) for the speed closed loop, we obtain the results exposed in Fig. 14 when requiring a set point rotor speed of 100 rad/s. There it can be verified that $Tr$ is 31.4 [ms] (considering again that IGBT’S are enabled at 1 [ms]). Using Equation 17 the bandwidth of the system would be 5.06 [Hz] which is again in practical terms the desired 5 [Hz] bandwidth.

Several simulations were performed to confirm a good speed response of the system. Proper results were obtained at steady...
state and during transients. In Fig. 15 it can appreciated a successful system response when 100 [rad/s] were required as a rotor speed reference from cero to 0.2 [ms] and then the reference changed to 40 [rad/s] until 0.4 [ms]. As it can be observed, the proposed control technique presents a suitable dynamic response which is a relevant advantage against schemes based on online search algorithms that show a poor dynamic behavior [15] [16] and undesirable torque disturbance [17]. In PSIM® the employed elements to perform the speed control are exposed in Fig. 16.

![Figure 12. System response without back-emf compensation](image1)

![Figure 13. System response with back-emf compensation](image2)

![Figure 14. Speed closed loop system response time](image3)

![Figure 15. Speed closed loop system response](image4)

![Figure 16. Speed control elements in PSIM®](image5)
5. CONCLUSIONS
In this paper, by means of a maximum torque per ampere strategy, a vector control for an IPMSM has been successfully implemented. The use of Park transformations was especially helpful as it permitted to track only two dc reference signals (Id and Iq) instead of three sinusoidal references. The proposed methodology has been validated by performing several simulation conditions in PSIM® software. Proper speed and torque control responses were attained on steady state and during transients. For the MTPA technique to be effective, nominal operating conditions should be given to the motor (mainly in terms of temperature) so that the parameters of the machine will not vary on time.

6. FUTURE WORK
As a part of future work, it is intended to consider non-linearities regarding the magnetic saturation and unwanted temperature effects of the motor that appear when working with higher current rates. Additionally, to improve the performance of the system, robust algorithms against unexpected parameter variations will be studied and used.

REFERENCES